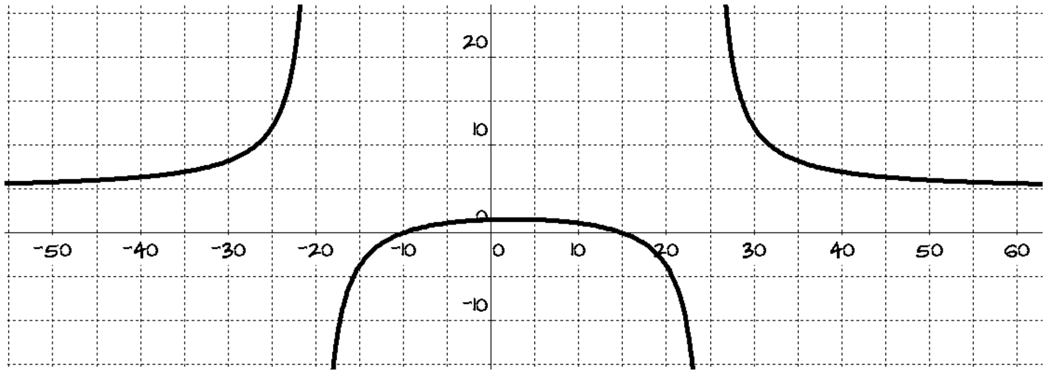
MATH 53 May 6, 2013

THIRD LONG EXAM Summer; AY 2012-2013

General Instructions: Use black or blue pen only. Show neat, complete and organized solutions to earn full points. Box all final answers. The use of any electronic devices is not allowed during the exam. Cheating is punishable by a grade of **5.00** for the course.

I. Write the letter of the best answer. Explain your answers briefly for partial points.

Let f be a continuous function **defined on $\mathbb{R}$** and suppose that the **graph of its derivative** is shown below. Use this to answer the first four items.



1. f is increasing on...

   A. $(-30,-20)$, but not $(-20,0)$

   B. $(-20,0)$, but not $(-30,-20)$

   C. both $(-30,-20)$ and $(-20,0)$

   D. neither $(-30,-20)$ nor $(-20,0)$

2. f is concave down on...

   E. $(-20,-10)$, but not $(30,45)$

   F. $(30,45)$, but not $(-20,-10)$

   G. both $(-20,-10)$ and $(30,45)$

   H. neither $(-20,-10)$ nor $(30,45)$

3. f has a relative extremum on...

   J. $x=-10$, but not on $x=25$

   K. $x=25$, but not on $x=-10$

   L. both $x=-10$ and $x=25$

   M. neither $x=-10$ nor $x=25$

4. f has a relative minimum on...

   N. $x=-10$, but not on $x=15$

   O. $x=15$, but not on $x=-10$

   P. both $x=-10$ and $x=15$

   Q. neither $x=-10$ nor $x=15$

MORE AT THE BACK

5. The graph of a polynomial function g is concave up on the interval $(-1, 1)$ if...

R. $g'(x)$ is increasing on $(-1, 1)$.

S. $g''(x) > 0$ on $(-1, 1)$.

T. Both are true.

U. Neither is true.

6. Let g be a continuous function. On which intervals is g guaranteed to have an absolute maximum and an absolute minimum?

W. $[-1, 1]$, but not $(-1, 1)$

X. $(-1, 1)$, but not $[-1, 1]$

Y. both $[-1, 1]$ and $(-1, 1)$

Z. neither $[-1, 1]$ nor $(-1, 1)$

II. Graph a function f defined on $\mathbb{R} \setminus \{0\}$ satisfying the following properties.

- The only zero of f is at $x = 3$.
- $\lim_{x\to\infty} f(x) = 1$
- $\lim_{x\to-\infty} f(x) = 1$
- $\lim_{x\to 0^-} f(x) = +\infty$
- $\lim_{x\to 0^+} f(x) = +\infty$
- f has exactly one relative extremum.
- f is increasing on the interval $(-\infty, 0)$.
- f is decreasing on the interval $(0, 3)$.
- f is increasing on the interval $(3, +\infty)$.
- Its only point of inflection is $(\frac{9}{2}, \frac{1}{9})$.
- f is concave up when $x < 0$.
- f is concave up when $0 < x < \frac{9}{2}$.

III. Consider $g(x) = \begin{cases} \dfrac{1}{x-1} & , x < 0 \\ \\ \dfrac{4x^2 + 4x + 6}{2x - 1} & , x \geq 0 \end{cases}$.

Find the **equations** of the three asymptotes of g.

Hint: One is horizontal, one is oblique and one is vertical.

IV. Let $h(x) = x^2 + x$. Show **using Mean Value Theorem** that there is a $c \in [0, \pi]$ such that $h'(c) = \pi + 1$. Verify that the assumptions of the said theorem are satisfied before using it.

V. Let $f(x) = \sin(2x) - x$.

Find the absolute minimum point and absolute maximum point of f on the interval $\left[0, \dfrac{\pi}{2}\right]$.

VI. Aaron's crush asks him to determine the **equation of the tangent line** to the graph of $g(x) = x^3 - x^2 - x + 1$ which has the least slope. Aaron has not taken Math 53 yet, but he could have found out the answer using an app in his iPad. But as we all know, he recently threw it from the top of a building. What is the answer to the Aaron's crush's question?

END OF EXAM

TOTAL: 12 POINTS

guissmo