MATH 54 19 May 2014

FIFTH LONG EXAM Summer; AY 2013-2014

Reminders: Answers written using anything other than black or blue ballpen may not be corrected. Items with insufficient or disorganized solutions may not gain full points. Insufficiently labelled graphs may not gain full points. Any form of cheating or academic dishonesty is subject to disciplinary action. Box your final answers.

I. Sketch the domain of $f(x) = \ln(x^2 + y^2 - 1) + \sqrt{4 - x^2 - 4y^2}$. Use solid lines for portions of the boundary in the domain, and dashed lines for portions not included. Use hollow points to emphasize points which are not included.

II. Evaluate the following limits.

1. Show that $\lim\limits_{(x,y)\to(0,1)} \dfrac{\cos x - y}{x + y - 1}$ does not exist.
2. Find $\lim\limits_{\substack{(x,y)\to(3,1) \\ x+y\neq 4}} \dfrac{\sqrt{x+y} - 2}{x + y - 4}$.

III. Suppose $w = 2x - z\tan^{-1} y$, $y = 3 - t$, $z = t$ and let x be defined implicitly as $\sin(tx) = t^2 + 2x$.

1. Use implicit differentiation to find dx/dt.
2. Find dw/dt.

IV. Find the equation of the plane tangent to the surface given by $f(x, y) = x^y + xy$ when $x = 2$ and $y = 3$.

V. The width, length and height of a box are 3 feet, 5 feet and 6 feet, respectively.

1. Use differentials to estimate the **change in volume**, dV, if the width is increased by 0.01 feet, the length is decreased 0.2 feet and the height is increased by 0.1 feet.
2. Use differentials to estimate the **change in surface area**, dS, if the width is increased by 0.01 feet, the length is decreased 0.2 feet and the height is increased by 0.1 feet. (Hint: The surface area is the sum of the areas of all six faces of the box.)

VI. Aaron, a bad guy, erased some parts of the blackboard while you were still copying the homework! Here are the following things you can still infer based on whats left.

- $f(x, y)$ is a function dependent on x and y.
- x is dependent on u and v.
- y is dependent on u and v.
- The local linear approximation of f at the point $(2, -3)$ is

$$L(x, y) = 1 + 4(x - 2) - 5(y + 3).$$

- At the point $(2, -3)$, $\dfrac{\partial f}{\partial u} = 18$ and $\dfrac{\partial y}{\partial u} = 2$.

Use these facts to answer the following questions

1. What is the value of $f(2, -3)$?
2. What is the value of $\dfrac{\partial x}{\partial u}$ at $(2, -3)$?

END OF EXAM.

"Hinabol pa ng Dobermaaaaan!" –Companero Brandy Ad