Math 54 — 12 May 2014

Fourth Long Exam — Summer; AY 2013-2014

Reminders: Answers written using anything other than black or blue ballpen may not be corrected. Items with insufficient or disorganized solutions may not gain full points. Insufficiently labelled graphs may not gain full points. Any form of cheating or academic dishonesty is subject to disciplinary action. Box your final answers.

I. Let $\mathbf{r}(t) = \left\langle \frac{1}{\sqrt{1-t^2}}, \quad \frac{1}{2t-1}, \quad \frac{\sin t - t}{t^3} \right\rangle$.

1. Find the natural domain of $\mathbf{r}$.
2. Find $\lim_{t\to 0} \mathbf{r}(t)$

II. Let $\mathbf{r}(t) = \left\langle \cos(e^t), \quad \sin(e^t), \quad e^\pi \right\rangle$.

1. Find $\mathbf{r}'(t)$.
2. Find the arclength parametrization $\mathbf{r}(s)$ of $\mathbf{r}(t)$. Use the point $(\cos 1, \sin 1, e^\pi)$.

III. Let $\mathbf{r}$ be a vector-valued function. Suppose that $\mathbf{r}(0) = \langle 0, 1, 0\rangle$, $\mathbf{r}'(0) = \langle 2, 0, 1\rangle$ and $\mathbf{r}''(0) = \langle 0, -4, 2\rangle$.

1. Find the curvature κ and the radius of curvature ρ at $t = 0$.
2. Find $\mathbf{N}(0)$, the unit normal vector at $t = 0$.

MORE AT THE BACK

IV. Let $\mathbf{r}(t)$ be the position function of the trajectory of a particle and let $\mathbf{v}(t) = \langle 4\sin(t), 2\cos(t/2), -3\rangle$ be its velocity vector.

1. If $\mathbf{r}(\pi) = \langle 4, 4, 4\pi\rangle$, find $\mathbf{r}(t)$.
2. Find a_T and a_N, the scalar tangential and scalar normal components of the acceleration vector at $t = 0$.

V. You have been transported to a location whose acceleration due to gravity is given by $-4m/s^2$. Aaron hovers over a point which is 48 meters in front of you. You launch a bottle containing the antidote for the zombie virus from the ground at a $45°$ angle with an initial speed of $12\sqrt{2}m/s$.

1. What is the maximum height of the bottle?
2. If the bottle hits Aaron, how high was he hovering?

VI. Do what is asked.

1. Find a parametric equation for the intersection of the elliptic paraboloid $4x^2 + y + 3z^2 = 1$ and the parabolic cylinder $y = z^2$.
2. Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be differentiable vector-valued functions and $\frac{d\mathbf{u}}{dt}, \frac{d\mathbf{v}}{dt}, \frac{d\mathbf{w}}{dt}$ be their respective derivatives. Show (step-by-step) that

$$\frac{d}{dt}\left(u \cdot v \times w\right) = \frac{d\mathbf{u}}{dt} \cdot \mathbf{v} \times \mathbf{w} + \mathbf{u} \cdot \frac{d\mathbf{v}}{dt} \times \mathbf{w} + \mathbf{u} \cdot \mathbf{v} \times \frac{d\mathbf{w}}{dt}.$$

END OF EXAM.

"E e o e o e e o e o." –Bastille