Reminders: Answers written using anything other than black or blue ballpen may not be corrected. Items with insufficient or disorganized solutions may not gain full points. Insufficiently labelled graphs may not gain full points. Any form of cheating or academic dishonesty is subject to disciplinary action. Box your final answers.

I. Let $\vec{a} = \langle 1, -2, 2 \rangle$, $\vec{b} = \langle 0, -4, 3 \rangle$ and θ be the angle between the vectors $\vec{a}$ and $\vec{b}$.

1. Find $\|\vec{a}\|$.
2. Find $\vec{b} \cdot \vec{b}$.
3. Find $\vec{a} \cdot \vec{b}$.
4. Find $\vec{a} \times \vec{b}$.
5. Find $\tan\theta$.
6. Find $\text{proj}_{\vec{b}}\,\vec{a}$.

II. Given $\langle 20, 21, 23 \rangle \cdot \langle 10, 11, 12 \rangle = 707$ and $\langle 20, 21, 23 \rangle \times \langle 10, 11, 12 \rangle = \langle -1, -10, 10 \rangle$.

1. Evaluate $\langle 60, 63, 69 \rangle \cdot \langle 30, 33, 36 \rangle$.
2. Evaluate $\langle 20, 22, 24 \rangle \times \langle 20, 21, 23 \rangle$.
3. Find a **unit vector** orthogonal to both $\langle 20, 21, 23 \rangle$ and $\langle 10, 11, 12 \rangle$.
4. Find the **area** of the triangle whose sides are given by the vectors $\langle 20, 21, 23 \rangle$ and $\langle 10, 11, 12 \rangle$.

III. Let $\ell_1 : \begin{cases} x = & t \\ y = & 1 \\ z = & 2 - t \end{cases}$ and $\ell_2 : \begin{cases} x = & 2 + t \\ y = & 3 + 4t \\ z = & 5 + t \end{cases}$.

1. Find a plane containing ℓ_1 whose normal vector is (also) orthogonal to ℓ_2.
2. Find the distance between the skew lines ℓ_1 and ℓ_2.

IV. Let $\ell_3 : \begin{cases} x = & 1 + 4t \\ y = & 5 - 2t \\ z = & 1 - 3t \end{cases}$ and $\ell_4 : \begin{cases} x = & t - 7 \\ y = & 2t - 1 \\ z = & t \end{cases}$.

1. Find the intersection of ℓ_3 and ℓ_4. (Hint: They intersect.)
2. Find the equation of the line parallel to ℓ_3 which passes through the center of the sphere $5x^2 + 5y^2 + 5z^2 + 20x - 6y - 8z + 24 = 0$.

V. Let $\ell : \begin{cases} x = & 2 \\ y = & 1 - t \\ z = & t \end{cases}$.

Aaron, a zombie manananggal who can draw circles, follows a path defined by ℓ. A photographer positioned at $(0, 0, 0)$ would like to take a picture of the rare creature.

1. How far is the photographer and the manananggal at the time when they are closest to each other?
2. When $t = 2$, how far will the photographer be from the manananggal?

VI. Let $Q : \dfrac{-x^2}{4} + 4y^2 + z^2 = 4$.

1. Give the equation, identify (give the name of the conic) and graph the traces of Q at each of the following planes.
   a. xy-plane
   b. xz-plane
   c. yz-plane
   d. $x = 4\sqrt{3}$
2. Identify (give the name) and graph the quadric Q.

END OF EXAM.

"Pika, pika, chu!" –Pikachu (#25)